

GENERAL RELATIVITY, DIFFERENTIAL GEOMETRY, AND UNITARY THEORIES IN THE WORK OF MIRA FERNANDES

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An analysis of the work of Mira Fernandes on unitary theories is presented.

1. Introduction

Aureliano de Mira Fernandes, born in 1884 in Portugal, was professor of Differential and Integral Calculus and Rational Mechanics, at Instituto Superior Técnico, from its foundation in 1911, onwards, until his retirement. He was also professor of Mathematical Analysis at the Instituto Superior de Ciências Económicas e Financeiras, what is now the Instituto Superior de Economia e Gestão. Mira Fernandes, by formation, was a mathematician. His Doctoral dissertation in 1911, supervised by Sidónio Pais, was on “Galois theory”, defended when he was 27 years old. From his dissertation to 1924 there are no publications. From 1924 onwards there are many publications, more than eighty, on several subjects, namely, group theory, differential geometry, unitary theories and rational mechanics. The most important papers were published in Rendiconti della Accademia dei Lincei, due to his friendship with Levi-Civita, the great Italian mathematical physicist. After Levi-Civita’s compulsory retirement, he published mainly in Portuguese journals. He also corresponded with Elie Cartan. Cartan in his work “Les espaces de Finsler” writes “It is after an exchange of letters with M. Aurelio (sic) de Mira-Fernandes, that I have perceived of the possibility of this simplification”. He was a member of the Lisbon Academy of Sciences from 1928 onwards. In 1932 he proposed Levi-Civita and Einstein to be foreign members of the Academy, a proposal accepted immediately by the President of the Academy, Egas Moniz, the future Nobel prize in medicine. These proposals were apt, since these two figures were pioneers in differential and Riemannian geometry, general relativity and unitary theories, areas for which Mira Fernandes devoted a great part of his scientific life (see¹ for the complete version of this article).

2. The scientific context of unitary theories

The idea of unification in physics is an old one, with Mie’s ideas being in the forefront in the 1912s. General relativity with its beautiful geometric structure, changed the picture from 1916 onwards. It put the gravitational field in a special relativity framework. However, it left electromagnetism out. Now, electricity and magnetism had been unified into electromagnetism using special relativity and a spacetime arena. Thus, one might argue, gravity (general relativity) and electromagnetism

should be unifiable in a unitary theory using a special world background as the new arena. This was advocated by many, notably by Eddington. What this world background could be was left imprecise. This rationale works if general relativity is a field theory, on the same footing of electromagnetism. But even this is controversial.

The first attempt to unify gravitation and electromagnetism was proposed by Weyl². In this theory the electromagnetic potential is introduced as a geometrical quantity which determines the transport law of a length scale. Weyl was able to reproduce Einstein's and Maxwell's equation within a single scheme. However, when confronted with observations the theory does not hold, atoms arriving at the earth from the cosmos would have different physical properties, as pointed out by Einstein. In spite of this demolishing problem, Weyl's idea of gauging was one of the most fruitful ideas in the history of physics in the context of quantum mechanics and electromagnetism. In any case the door to unification schemes was open. There is a Brazilian saying that says "Where one ox passes a herd of oxen passes". It applies neatly here. The next ox to pass was Eddington's theory³. Eddington set forward the idea that, perhaps, the connection $\Gamma_{\mu\nu}^{\lambda}$ is the primary quantity, rather than the metric $g_{\mu\nu}$ itself and partially developed a theory. Einstein in between the years of 1923 and 1925 fiddled with the theory but could not progress.

Further ideas on connections and parallel transport from mathematicians were also used to propose physical theories of gravitation and electromagnetism. A general connection Γ (dropping the indices) has a metric part, a homothetic part as in Weyl's theory, and a torsion, which is the antisymmetric part of the connection. Thus, besides the Riemann-Christoffel curvature, one gets a homothetic curvature, and a torsion curvature. Unitary theories, that tried to unify the gravitational and electromagnetic fields used one or all these connections and curvatures. To see some of these, see⁴ for reviews and precise citations, see also¹. Another idea on connections that sprang at the time, and is seldom mentioned, is that the manifold can see a connection Γ for contravariant vectors v and a different connection Γ' for covariant vectors u . Thus, for each connection, Γ and Γ' , one gets the usual Riemann-Christoffel curvature, a torsion curvature, and a homothetic curvature. These two connections give rise to a new three-index tensor field C which in turn makes the bridge between the connections themselves. The field C is defined as the covariant derivative of the identity tensor I . In most theories, this C field was put to zero, probably because of its apparent lack of physical meaning. As we will see, not for Mira Fernandes. In the years between 1926 and 1933 he explored some of the proposed theories by adding to them the C field, while trying to physically interpret it.

3. The works of Mira Fernandes on unitary theories of gravitation and electromagnetism

The book "Foundations of differential geometry of the linear spaces" (in Portuguese), 1926 is based on Schouten's book of 1924 "Ricci-Kalkül" (in German) and displays the theory of connections at the time. A paper of 1931 in Rendiconti

shows some seven new properties of connections.

In the paper of 1932⁵ Mira Fernandes ventures into unitary theories. He analyzes Straneo's papers, an Italian mathematical physicist from the group of Levi-Civita. Straneo published papers on unitary theories, see his review⁶. Straneo's connection that most interests Mira Fernandes is $\Gamma_{\beta\gamma}^{\alpha} = \{\frac{\alpha}{\beta\gamma}\} + (\delta_{\mu}^{\alpha}\psi_{\nu} - \delta_{\nu}^{\alpha}\psi_{\mu})$, where ψ_{ν} is an additional vector field of the theory, to be equated physically to the electromagnetic potential. This equation is based on Weyl's connection and generalizes it. However, it does not have the same mathematical and physical substrata. Mira Fernandes supposes that his own connection is invariant by incidence, i.e., $C_{\alpha\beta}{}^{\gamma} = C_{\alpha}{}^{\gamma}{}_{\beta}$, covariant symmetric, and contravariant metric. The aim of his Note is to formulate considerations about Straneo's connection, involving the field C_{α} . For a four-dimensional spacetime, $n = 4$, the connection, in Mira Fernandes' words, satisfies "all the conditions attributed by Straneo to the structure of the physical space". The vector ψ_{α} representing the electromagnetic potential is such that $\psi_{\alpha} = -\frac{1}{2}C_{\alpha}$, and the torsion S and nonmetricity Q are given by $S_{\alpha} = \frac{1}{2}Q_{\alpha} = C_{\alpha}$. The field C provides much of the physical and geometrical content of the theory. It is also noted that the connection "is distinct from Weyl's since $C \neq 0$ ". When $C_{\alpha} = C_{,\alpha}$, i.e., is a gradient, then one recovers Weyl. There are other related publications, see¹.

4. Conclusions

Theories which change Riemannian geometry, as those used by Mira Fernandes, are not in fashion as theories of unification, they were reverted to theories of gravitation and spin, the Einstein-Cartan theories. Theories that use extra dimensions to obtain fields in four dimensions, the Kaluza-Klein theories, were not touched by Mira Fernandes. These ideas are still in use in supergravity and string theory. The name of such theories has been changing, unitary theories at first, then unified field theories, and nowadays theories of everything. Will their fate be the same as Mie's theory?

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